# IMO 2025 Problems with Solution Outlines

## Part 1 · The paper at a glance

66th IMO | Sunshine Coast, Australia | July 2025

**CHART** Six problems, two days, 4 hours 30 minutes per day, 7 points per problem (42 in total).

| P1 | P2 | P3 | P4 | P5 | P6 |
|---|---|---|---|---|---|
| **Combinatorics** | **Geometry** | **Number theory** | **Number theory** | **Algebra** | **Combinatorics** |
| C1 · Find the answer | G4 · Proof | N7 · Find the answer | N3 · Find the answer | A3 · Find the answer | C8 · Find the answer |
| Day 1 | Day 1 | Day 1 | Day 2 | Day 2 | Day 2 |

## A Day 1 · Problems 1 to 3

### 1 Problem 1

Combinatorics · C1

Proposed by Linus Tang, USA

A line in the plane is called sunny if it is not parallel to the x-axis, the y-axis or the line $x + y = 0$. Let $n \geq 3$. Find all non-negative integers $k$ for which there are $n$ different lines such that (1) every point $(a, b)$ with positive integers $a$, $b$ and $a + b \leq n + 1$ lies on at least one of the lines, and (2) exactly $k$ of the lines are sunny.

**ANSWER** **$k = 0$, 1 or 3.**

**KEY IDEA**

The points form a triangle with $3n - 3$ points on its boundary. For $n \geq 4$, $n$ lines that each meet the boundary in at most two points cover at most $2n < 3n - 3$ of them, so one of the lines must be a whole side of the triangle ($x = 1$, $y = 1$ or $x + y = n + 1$). That line is not sunny; removing it leaves the same problem for $n - 1$. This reduces everything to $n = 3$, where checking the cases gives $k = 0, 1, 3$ and rules out $k = 2$.

Full written solutions: AoPS wiki, 2025 IMO Problem 1

### 2 Problem 2

Geometry · G4

Proposed by Tran Quang Hung, Vietnam

Circles $\Omega$ and $\Gamma$ have centres $M$ and $N$, and the radius of $\Omega$ is smaller than that of $\Gamma$. They meet at two points $A$ and $B$. Line $MN$ meets $\Omega$ at $C$ and $\Gamma$ at $D$, with $C$, $M$, $N$, $D$ in this order. $P$ is the circumcentre of triangle $ACD$. Line $AP$ meets $\Omega$ again at $E$ and $\Gamma$ again at $F$. $H$ is the orthocentre of triangle $PMN$. Prove that the line through $H$ parallel to $AP$ is tangent to the circumcircle of triangle $BEF$.

**PROOF PROBLEM**

**Problem 2, continued**

**KEY IDEA**

An angle chase gives CE ∥ AD and DF ∥ AC, so lines CE and DF meet at a point A′ with ACA′D a parallelogram, and BA′ ∥ CD. Let T be the circumcentre of triangle A′EF. Angle chasing shows that T lies on line BA′ and on the circumcircle of BEF, with TE = TF, so T is the midpoint of arc EF; T also lies on lines ME and NF. Since PN and PM are the perpendicular bisectors of AD and AC, MH ∥ AD and NH ∥ AC, and from this H is the incentre of triangle TMN. Comparing angles then gives HT ∥ AP. So the line through H parallel to AP passes through T and is parallel to chord EF, which makes it the tangent at the arc midpoint T.

Full written solutions: AoPS wiki, 2025 IMO Problem 2

## 3 Problem 3

Number theory · N7

Proposed by Lorenzo Sarria, Colombia

A function f from the positive integers to the positive integers is called bonza if f(a) divides $b^a - f(b)^{f(a)}$ for all positive integers a and b. Find the smallest real constant c such that f(n) ≤ cn for all bonza functions f and all positive integers n.

**ANSWER** **c = 4.**

**KEY IDEA**

Taking a = b = n shows f(n) divides $n^n$. Looking at primes: either f(p) > 1 for infinitely many primes p, which forces f(n) = n for all n, or f(p) = 1 for all large primes. In the second case f(n) is 1 for odd n and a power of 2 for even n, and a careful count of powers of 2 gives f(n) ≤ 4n. A bonza function with f(4) = 16 shows that 4 cannot be improved.

Full written solutions: AoPS wiki, 2025 IMO Problem 3

# B Day 2 · Problems 4 to 6

## 4 Problem 4

Number theory · N3

Proposed by Paulius Aleknavičius, Lithuania

A proper divisor of a positive integer N is a positive divisor of N other than N. An infinite sequence $a_1, a_2, \ldots$ of positive integers is such that each term has at least three proper divisors and $a_{n+1}$ is the sum of the three largest proper divisors of $a_n$. Find all possible values of $a_1$.

**ANSWER** **$a_1 = 6 \cdot 12^e \cdot m$, where $e \geq 0$ and m is coprime to 10.**

**Problem 4, continued**

**KEY IDEA**

If N is divisible by 6 but not by 4 or 5, its three largest proper divisors are N/2, N/3, N/6, which add up to N: the sequence stays fixed. If N is divisible by 12, they are N/2, N/3, N/4 and the next term is 13N/12. So $a_1 = 6 \cdot 12^e \cdot m$ climbs e times and then stays fixed. For the converse: an odd term is followed by a smaller odd term, and an even term not divisible by 3 is followed by a smaller term that is again not a multiple of 6. Either way the sequence would decrease for ever, so every term is a multiple of 6. A multiple of 6 divisible by 5 but not by 4 would be followed by 31N/30, which is odd, so that never happens. Hence each step either multiplies by 13/12 or changes nothing, and 13/12 can be applied only finitely often.

Full written solutions: AoPS wiki, 2025 IMO Problem 4

## 5 Problem 5

Algebra · A3

Proposed by Massimiliano Foschi and Leonardo Franchi, Italy

Alice and Bazza play a game that depends on a positive real number λ. On turn n (n = 1, 2, ...): if n is odd, Alice chooses a real $x_n \geq 0$ with $x_1 + \ldots + x_n \leq \lambda n$; if n is even, Bazza chooses a real $x_n \geq 0$ with $x_1^2 + \ldots + x_n^2 \leq n$. A player who cannot choose loses; if the game never ends nobody wins. Find all λ for which Alice has a winning strategy and all λ for which Bazza has one.

**ANSWER** **Alice wins exactly when $\lambda > 1/\sqrt{2}$. Bazza wins exactly when $\lambda < 1/\sqrt{2}$. For $\lambda = 1/\sqrt{2}$ neither can force a win.**

**KEY IDEA**

Bazza's plan: always pick $x_{2k}$ so that the sum of squares is exactly 2k, that is $x_{2k} = \sqrt{(2 - x_{2k-1}^2)}$. Then $x_{2k-1} + x_{2k} \geq \sqrt{2}$, so the running sum after 2k turns is at least $k\sqrt{2}$, and Alice runs out of room when $\lambda < 1/\sqrt{2}$. Alice's plan: play 0 for many turns. Bazza's k numbers have squares adding up to at most 2k, so their sum is at most $k\sqrt{2}$, and her spare room $\lambda(2k + 1) - k\sqrt{2}$ grows without bound when $\lambda > 1/\sqrt{2}$. Once it exceeds $\sqrt{(2k + 2)}$ she plays a number larger than $\sqrt{(2k + 2)}$, the sum of squares passes 2k + 2, and Bazza cannot move. For $\lambda = 1/\sqrt{2}$ the same two plans show that neither player can be forced to lose.

Full written solutions: AoPS wiki, 2025 IMO Problem 5

## 6 Problem 6

Combinatorics · C8

Proposed by Zhao Yu Ma and David Lin Kewei, Singapore

A 2025 × 2025 grid of unit squares is given. Matilda places rectangular tiles, possibly of different sizes, with sides on grid lines and no unit square covered twice. Find the smallest number of tiles she needs so that every row and every column has exactly one unit square that is not covered.

**ANSWER** **2112.**

**Problem 6, continued**

**KEY IDEA**

The uncovered squares form a permutation pattern. For a grid of side $k^2$ the answer is $k^2 + 2k - 3$; here $k = 45$ gives $2025 + 90 - 3 = 2112$. The construction places the uncovered squares on a tilted $k \times k$ lattice so that large tiles fit between them. For the lower bound, take a longest increasing run (length $a$) and a longest decreasing run (length $b$) among the uncovered squares; by the Erdős–Szekeres theorem $ab \geq 2025$, so $a + b \geq 90$. The two runs split the board into north, east, south and west regions. For every uncovered square, mark the cell next to it on the outward side of each region it belongs to. No tile can contain two marked cells and at most four marks fall off the board; counting the marks gives at least $2025 + a + b - 3 \geq 2112$ tiles.

Full written solutions: AoPS wiki, 2025 IMO Problem 6

## How to use this sheet

Try each problem before you read the key idea. The key idea is a sketch of one route to the solution, not a full proof: use it as a hint, then write the complete argument yourself and compare it with a full written solution.

The problems on this sheet are restated in Gonit's own words. For the official wording, download the IMO 2025 paper from imo-official.org. Full solutions: AoPS wiki, 2025 IMO and Evan Chen's IMO 2025 solution notes.