IMO 2026 Problems with Solution Outlines

Part 1 · The paper at a glance

67th IMO | Shanghai, China | July 2026

CHART **Six problems, two days, 4 hours 30 minutes per day, 7 points per problem (42 in total).**

P1	P2	P3	P4	P5	P6
Number theory	Geometry	Combinatorics	Combinatorics	Algebra	Number theory
Proof Day 1	Proof Day 1	Find the answer Day 1	Find the answer Day 2	Find the answer Day 2	Proof Day 2

Topic labels for 2026 are Gonit's own. The official shortlist codes are published after the next IMO.

A Day 1 · Problems 1 to 3

1 Problem 1

Number theory

Proposed by Giancarlo Kerg, Luxembourg

2026 integers greater than 1, not necessarily different, are written on a board. In a move, Confucius picks two numbers $m > 1$ and $n > 1$ from different places and replaces them with $\gcd(m, n)$ and $\operatorname{lcm}(m, n)/\gcd(m, n)$. He keeps making moves while he can. (a) Prove that, whatever he chooses, after finitely many moves exactly one number M on the board is greater than 1. (b) Prove that M does not depend on his choices.

PROOF PROBLEM

KEY IDEA

(a) If $\gcd(m, n) = 1$ the move creates a new 1, and a 1 can never be picked again. If $\gcd(m, n) > 1$ the product of all the numbers gets smaller. Neither can happen for ever, and a move never leaves two 1s, so the process ends with exactly one number above 1. (b) Fix a prime p. If m and n contain p to the powers $x \le y$, the new numbers contain it to the powers x and $y - x$. This is one step of the Euclidean algorithm, so the gcd of all the exponents of p never changes. At the end that gcd is the exponent of p in M.

Full written solutions: AoPS wiki, 2026 IMO Problem 1

2 Problem 2

Geometry

Proposed by Mykhailo Shtandenko, Ukraine

In triangle ABC, M and N are the midpoints of AB and AC. Points K and L are chosen inside triangles BMC and BNC so that K is strictly inside triangle LBA and L is strictly inside triangle ACK. Suppose $\angle KBA = \angle ACL$, $\angle LBK = \angle LNC$ and $\angle LCK = \angle BMK$. O is the circumcentre of triangle AKL. Prove that $OM = ON$.

PROOF PROBLEM

Problem 2, continued

KEY IDEA

Let BK meet AC at X and CL meet AB at Y. Each angle condition says four points are concyclic: B, Y, X, C (and so M, Y, X, N, because MN ∥ BC); X, N, L, B; and Y, M, K, C. Repeated use of the power of a point shows that A, K, L and the midpoints of BY and CX lie on one circle, and that M and N have the same power with respect to this circle. Equal power means equal distance from its centre O.

Full written solutions: AoPS wiki, 2026 IMO Problem 2

3 Problem 3

Combinatorics

Proposed by Ilya Bogdanov and Grigorii Chelnokov, Russia

Let n be a positive integer. Liu Bang and Xiang Yu share a stick of length 1. Liu Bang marks at most n points on it, then Xiang Yu marks at most n more points; all marked points are different. The stick is cut at every marked point. The players then take turns claiming one unclaimed piece, Liu Bang first, each trying to maximise the total length of his own pieces. For each n, find the largest c such that Liu Bang can be sure of total length at least c.

ANSWER $c = 2^n/(2^{n+1} - 1)$.

KEY IDEA

With the pieces sorted from longest to shortest, the first player takes the 1st, 3rd, 5th, … pieces. Liu Bang's best marking cuts the stick in the ratio $1 : 2 : 4 : \ldots : 2^n$. For the other direction, Liu Bang's n + 1 segments have 2^{n+1} subset sums between 0 and 1, so by the pigeonhole principle two disjoint groups of segments differ in total length by at most $1/(2^{n+1} - 1)$. Xiang Yu cuts so that the pieces of the two groups pair off equally and bisects everything else, which holds Liu Bang's lead to that difference.

Full written solutions: AoPS wiki, 2026 IMO Problem 3

B Day 2 · Problems 4 to 6

4 Problem 4

Combinatorics

Proposed by Switzerland

Shan-Yu and Mulan play a game with an angle θ, where $0° < \theta < 180°$, known to both. Shan-Yu first makes a paper triangle T of his choice. Then they repeat: if T has an angle exactly θ, Mulan wins. Otherwise Mulan picks a point P on the perimeter of T that is not a vertex and cuts from P to the opposite vertex, making two triangles; Shan-Yu throws one away and the other becomes the new T. For which θ can Mulan be sure to win in finitely many steps?

ANSWER Exactly when $\theta = 180°/n$ for an integer $n \geq 2$.

Problem 4, continued

KEY IDEA

If $\theta = 180°/n$: any triangle with an angle equal to a multiple $k\theta$ is winning, because Mulan can cut off θ from that angle step by step. She first cuts along an altitude to get a right triangle, then makes one more cut that creates an angle $k\theta$ between 45° and 90°. If θ is not of this form, call an angle safe when it is not a multiple of θ. A triangle with three safe angles cannot be cut into two triangles that both contain an unsafe angle, so Shan-Yu starts with an all-safe triangle and always keeps an all-safe piece.

Full written solutions: AoPS wiki, 2026 IMO Problem 4

5 Problem 5

Algebra

Proposed by Alfrēds Saročinskis, Latvia

Find all functions f from the positive real numbers to the positive real numbers such that $\sqrt{((x^2 + f(y)^2)/2)} \geq (f(x) + y)/2 \geq \sqrt{(x \cdot f(y))}$ for all positive real x and y.

ANSWER **$f(x) = x + c$ for any constant $c \geq 0$.**

KEY IDEA

These functions work by the inequality between the quadratic, arithmetic and geometric means of x and y + c. Conversely, put x = f(t) and y = t: both ends of the chain become f(t), so (f(f(t)) + t)/2 = f(t). Hence t, f(t), f(f(t)), … is an arithmetic progression for every t, with common difference at least 0. Comparing two such progressions through the right-hand inequality shows that all positive common differences are equal, and a final check removes the mixed case where some points are fixed and others are not.

Full written solutions: AoPS wiki, 2026 IMO Problem 5

6 Problem 6

Number theory

Proposed by Hung-Hsun Hans Yu, Taiwan

$a_1, a_2, a_3, \ldots$ is an infinite sequence of integers greater than 1. For every n, a_{n+1} is the smallest integer greater than a_n that shares a common factor greater than 1 with each of $a_1, a_2, \ldots, a_n$. Prove that there are positive integers T and L such that $a_{n+T} = a_n + L$ for every n.

PROOF PROBLEM

KEY IDEA

Only finitely many primes matter. Example: starting from 15 the sequence is 15, 18, 20, 24, 30, 36, 40, 42, 45, …, which is every integer from 15 on that is divisible by 6, 10 or 15. In general, one proves that a term containing a prime larger than a_1^2 is never needed as a test for later terms. So whether a number belongs to the sequence depends only on its remainder modulo L, the product of the primes up to a_1^2, and T is the number of members in each block of L consecutive integers.

Full written solutions: AoPS wiki, 2026 IMO Problem 6

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How to use this sheet

Try each problem before you read the key idea. The key idea is a sketch of one route to the solution, not a full proof: use it as a hint, then write the complete argument yourself and compare it with a full written solution.

The problems on this sheet are restated in Gonit's own words. For the official wording, download the IMO 2026 paper from imo-official.org. Full solutions: AoPS wiki, 2026 IMO and Evan Chen's IMO 2026 solution notes.